

FORTY-EIGHTH ANNUAL AUGUSTA
UNIVERSITY MATHEMATICS CONTEST
MARCH 3, 2023
TEAM PROBLEMS

1. If x, y , and z are real numbers exceeding 1, and w is a positive real number such that

$$\log_x w = 24, \log_y w = 40, \text{ and } \log_{xyz} w = 12,$$

then find the value of $\log_z w$.

2. A group of friends went to a restaurant for lunch. They had agreed to split the bill equally. However, when the bill arrived, two of them discovered that they had left their money at home. The others in the group then agreed to make up the difference, resulting in each one having to pay an extra \$1.30. If the total bill was \$78.00, how many people were in the group that had lunch at the restaurant?
3. In how many different ways can the number 12 be written as a sum of 1's and 2's if the order of the summands is considered? For example, there are five ways to write the number 4, namely

$$1 + 1 + 1 + 1, 1 + 1 + 2, 1 + 2 + 1, 2 + 1 + 1, \text{ and } 2 + 2.$$

4. If m is chosen randomly from the set $\{0, 1, 2, 3\}$, and (independently) n is chosen randomly from the set $\{0, 1, 2\}$, what is the probability that the equation $x^2 + 2mx + n^2 = 0$ has one or more real roots?
5. Let $f(x)$ be a function such that, for every real number x ,

$$f(x) + 2f(-x) = \sin x.$$

What is the value of $f\left(\frac{\pi}{2}\right)$?

6. Find the equation of the parabola which passes through the points $(-1, 4)$, $(1, 0)$, and $(2, 16)$. Write your answer in the form $y = ax^2 + bx + c$.